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Tidal streams in a MOND potential: constraints from Sagittarius

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ABSTRACT

We compare orbits in a thin axisymmetric disc potential in Modified Newtonian Dynamics (MOND) with those in a thin disc plus near-spherical dark matter halo predicted by a ΛCDM cosmology. Remarkably, the amount of orbital precession in MOND is nearly identical to that which occurs in a mildly oblate CDM Galactic halo (potential flattening $q = 0.9$), consistent with recent constraints from the Sagittarius stream. Since very flattened mass distributions in MOND produce rounder potentials than in standard Newtonian mechanics, we show that it will be very difficult to use the tidal debris from streams to distinguish between a MOND galaxy and a standard CDM galaxy with a mildly oblate halo.

If a galaxy can be found with either a prolate halo or one that is more oblate than $q \sim 0.9$ this would rule out MOND as a viable theory. Improved data from the leading arm of the Sagittarius dwarf – which samples the Galactic potential at large radii – could rule out MOND if the orbital pole precession can be determined to an accuracy of the order of $\pm 1^{\circ}$.

Key words: galaxies: haloes – galaxies: kinematics and dynamics – cosmology: theory.

1 INTRODUCTION

Ever since Zwicky's seminal work in the 1930s it has been known that there is a disparity between the mass of galaxies as measured dynamically and the mass inferred from the visible light. The standard explanation for this missing matter is to invoke one or many weakly interacting massive particles that formed early in the Universe and that to first order interact only via gravity (see e.g. Bergström 1998). This is known as cold dark matter or CDM theory. However, since none of the candidate particles has yet been detected, it is important also to consider alternative theories as an explanation for the missing matter.

One such alternative theory is a modification of standard Newtonian gravity (called Modified Newtonian Dynamics, or MOND) for accelerations below some characteristic scale, $a_0 \sim 1.2 \times 10^{-10}\ \mathrm{ms}^{-2}$ (Milgrom 1983; McGaugh 2004). MOND was first suggested by Milgrom in 1983 as a modified inertia theory, but since then has been expanded into a self-consistent Lagrangian field theory (Bekenstein & Milgrom 1984) and, more recently, has been placed on a firm footing within the context of general relativity (Bekenstein 2004). This last point is of particular interest since Bekenstein (2004) has managed to address many of the conceptual problems that have plagued MOND over the past two decades and has shown that the theory can be consistent with gravitational lensing and other general-relativistic phenomena.

In this paper we compare the potential of the Milky Way as predicted by MOND and CDM models. In the former the potential arises from a flattened disc of baryons, whereas the latter potential is primarily from an extended spheroidal distribution of dark matter. A perfectly spherical potential has orbits that are confined to lie on planes (Binney & Tremaine 1987). By contrast, orbits in axisymmetric potentials generally show precession of their orbital planes (this will be true for all orbits that are not exactly planar or exactly polar).

Ibata et al. (2001) and more recently Majewski et al. (2003), Johnston, Law & Majewski (2005) and Law, Johnston & Majewski (2005) have studied the tidal debris from the Sagittarius dwarf galaxy and calculated likely orbits for the galaxy and its stellar debris. They find that the precession of the orbital plane is small ($\sim 10^{\circ}$) and is consistent with a Galactic halo potential that is only mildly oblate ($q = 0.9$–0.95) (although see also Helmi 2004 and Section 5 in this paper). In the MOND model, where all of the gravity comes from the disc, the potential may be much flatter, leading to far more precession than is observed. In this way, tidal debris from infalling satellites such as the Sagittarius dwarf galaxy could provide strong constraints on any altered gravity theory that supposes that all gravity is produced only by the visible light.

This paper is organized as follows. In Section 2 we discuss the Galactic MOND potential. In Section 3 we outline the initial conditions and orbit solver. We use four models: the CDM model, which has a spherical dark matter halo, the f095CDM and f09CDM models, which have slightly oblate haloes ($q = 0.95$ and $q = 0.9$), and

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the MOND model, in which all of the gravitational potential comes from the disc. In Section 4 we present our results for two orbits: the first is motivated by the orbit of the Sagittarius dwarf, and the second is a small-pericentre orbit chosen to sample a wide range of the Galactic potential. In Section 5 we briefly discuss the significance of these results and relate our work to previously published studies. Finally, in Section 6 we present our conclusions.

2 THE MOND POTENTIAL

The MOND field equations lead to a modified, non-linear, version of Poisson's equation given by (Bekenstein & Milgrom 1984)

$$\nabla \cdot [\mu(|\nabla(\Phi)|/a_0)\nabla\Phi] = 4\pi G\rho, \tag{1}$$

where ρ is the density, Φ is the scalar field for MOND gravity, and a_0 is the acceleration scale below which gravity deviates from standard Newtonian behaviour. The unknown function $\mu(|\nabla(\Phi)|/a_0)$ parametrizes the change from Newtonian to MOND gravity and is usually given phenomenologically by $\mu(x) = x(1 + x^2)^{-1/2}$ (Bekenstein & Milgrom 1984).

Equation (1) is in general extremely difficult to solve, not least because it is trivial to show that making the substitution $\Phi \rightarrow \Phi_1 + \Phi_2$ does not give $\rho \rightarrow \rho_1 + \rho_2$. This means that solutions cannot be superposed as in normal Newtonian mechanics. Every mass configuration will have its own unique potential which should be determined by (numerically) inverting equation (1). Thus, while some authors (see e.g. Knebe & Gibson 2004) have made valiant efforts to adapt *N*-body integrators to work in MOND, these can only be, at best, approximations.

Equation (1) can be solved, however, in extremely special cases. Following Brada & Milgrom (1995), notice that we can write the MOND gravitational field ($\boldsymbol{g} = \nabla\Phi$) as the sum of the Newtonian gravitational field ($\boldsymbol{g}_\mathrm{N} = \nabla\Phi_\mathrm{N}$) and a curl field:

$$\mu(|\boldsymbol{g}|/a_0)\boldsymbol{g} = \boldsymbol{g}_\mathbf{N} + \nabla \times \boldsymbol{h}. \tag{2}$$

The curl field will trivially vanish for planar, spherical or cylindrical symmetry, giving (in exact agreement with the modified inertia interpretation of MOND, see Milgrom 1983)

$$\mu(|\boldsymbol{g}|/a_0)\boldsymbol{g} = \boldsymbol{g}_\mathbf{N}, \tag{3}$$

which, substituting for $\mu(x) = x(1 + x^2)^{-1/2}$ as above and inverting gives

$$\boldsymbol{g} = \boldsymbol{g}_\mathbf{N}\frac{\left(1 + \sqrt{1 + 4a_0^2/|\boldsymbol{g}_\mathbf{N}|^2}\right)^{1/2}}{\sqrt{2}}. \tag{4}$$

Equation (4) is much more tractable since $\boldsymbol{g}_\mathrm{N}$ can be calculated from the Newtonian potential as usual and then simply modified to give the correct MOND acceleration at a given point in the field.

Our Galaxy is clearly neither planar, spherical nor cylindrical, and so the applicability of equation (4) may rightly be questioned. However, Brada & Milgrom (1995) demonstrated that equation (4) can be used *exactly* for infinitesimally thin Kuzmin discs with *Newtonian* potential given by (Binney & Tremaine 1987)

$$\Phi_\mathrm{N}(R, z) = \frac{-GM}{\sqrt{R^2 + (a + |z|)^2}}, \tag{5}$$

where G is the gravitational constant, a is the disc scalelength and M is the mass of the disc. The reason that the Kuzmin potential can be used exactly is because it is an extremely special potential for which $|\nabla\Phi_\mathrm{N}| = f(\Phi_\mathrm{N})$.[1] Notice that in MOND, the force field is still the gradient of a scalar potential and so the MOND field must be conservative; that is, $\nabla\times\boldsymbol{g} = \mathbf{0}$. Thus a MOND field can be generated via equation (3) from a Newtonian field provided that the Newtonian field satisfies the following constraint: $\nabla|\nabla\Phi_\mathrm{N}| \times \nabla\Phi_\mathrm{N} = \mathbf{0}$. This is satisfied exactly by the Kuzmin disc.

In this paper we use the Kuzmin potential to study orbits in axisymmetric potentials in MOND. We compare these orbits with similar orbits in standard Newtonian mechanics (the CDM model) using the Kuzmin potential plus a flattened spherical logarithmic potential (to model the dark matter) given by (Binney & Tremaine 1987)

$$\Phi_\mathrm{L}(R, z) = \frac{1}{2}v_0^2 \ln\left(R_\mathrm{c}^2 + R^2 + \frac{z^2}{q^2}\right) + \mathrm{constant}, \tag{6}$$

where R_c is the scalelength, $0.7 \leqslant q \leqslant 1$ is the halo flattening, and v_0 is the asymptotic value of the circular speed of test particles at large radii in the halo.

We will also compare these with the more realistic Galactic potential used by Johnston et al. (2005) and Law et al. (2005). They use a logarithmic halo (equation 6), a Miyamoto–Nagai potential for the disc (Binney & Tremaine 1987), and a Hernquist potential for the bulge (Hernquist 1990):

$$\Phi_\mathrm{disc} = \frac{-GM}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}}, \tag{7}$$

$$\Phi_\mathrm{bulge} = \frac{-GM_\mathrm{bulge}}{r + c}, \tag{8}$$

where a is the disc scalelength as in equation (5), b is the disc scaleheight, M is the disc mass, c is the bulge scalelength and M_bulge is the bulge mass. Notice that for $b \rightarrow 0$ equation (7) reduces to the Kuzmin disc in equation (5).

3 INITIAL CONDITIONS AND ORBIT SOLVING

The mass distribution we use in MOND is the flattened Kuzmin disc (see equation 5). For the CDM model we use a Kuzmin disc plus a logarithmic halo (see equation 6). We present three CDM models: one with no halo flattening (CDM), one with $q = 0.95$ (f095CDM) and one with $q = 0.9$ (f09CDM). We compare these with the best-fitting Milky Way potential from Johnston et al. (2005) and Law et al. (2005) (L05). The parameters used in all five models are given in Table 1 and are chosen to match the measured rotation curve of the Milky Way.

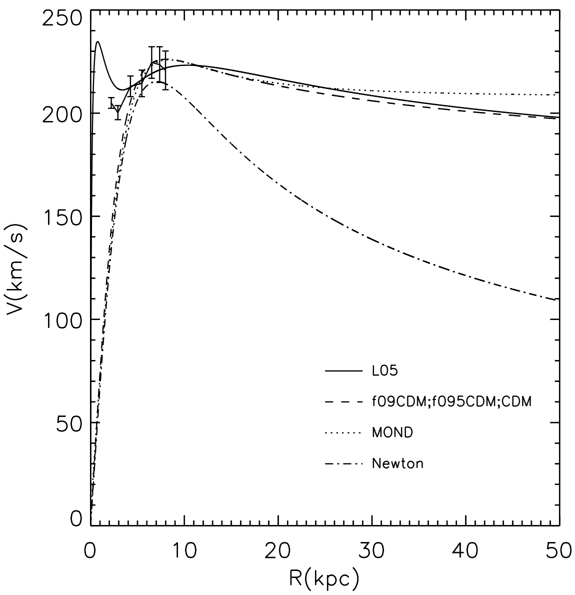
Fig. 1 shows the rotation curve for the MOND model (dotted line), the CDM models[2] (dashed line) and the L05 model (solid line). The rotation curve that the Kuzmin disc would give *without* MOND is overplotted for comparison (dot-dashed line). The black data points show the mean of H I measurements of the rotation curve taken from Bania & Lockman (1984), Weaver & Williams (1974, 1973), Malhotra (1995) and Kerr et al. (1986). It is important to note that we are not attempting to form an accurate model of the Milky Way in this paper; rather, we wish only to compare orbits in MOND

[1] For the Kuzmin potential, $|\nabla\Phi_\mathrm{N}| = \Phi^2{}_\mathrm{N}/GM$.

[2] All of the CDM models will produce the same rotation curve since the force from the Galaxy on the satellite *in the plane* of the Galaxy is independent of the dark matter halo flattening, q.

Table 1. Initial conditions.

Model	$M(\mathrm{M}_\odot)$	a(kpc)	b(kpc)	$v_0(\mathrm{km\,s^{-1}})$	R_c(kpc)	q	$M_{\mathrm{bulge}}(\mathrm{M}_\odot)$	c(kpc)	$a_0(\mathrm{m s^{-2}})$
MOND	1.2×10^{11}	4.5	–	–	–	–	–	–	1.2×10^{-10}
CDM	1.2×10^{11}	4.5	–	175	13	1	–	–	–
f095CDM	1.2×10^{11}	4.5	–	175	13	0.95	–	–	–
f09CDM	1.2×10^{11}	4.5	–	175	13	0.9	–	–	–
L05	1×10^{11}	6.5	0.26	171	13	0.9	3.53×10^{10}	0.7	–



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and CDM galaxies. The Kuzmin potential is not the most accurate model for the stellar distribution of the Milky Way (see e.g. Caldwell & Ostriker 1981), and in both the CDM and the MOND model we have made no attempt to model the stellar bulge and bar although it is well known that they contribute significantly to the potential of the Galaxy (Caldwell & Ostriker 1981; Dwek et al. 1995). This can be seen in the difference between the L05 rotation curve with a bulge component (solid line) and all of the other models. Interior to $\sim$10 kpc, all of the rotation curves deviate quite strongly from L05. Since most Milky Way satellites orbit well outside 10 kpc, however, a potential model for the Milky Way that is accurate beyond this point should suffice.

The equation of motion in the MOND and CDM models is given by

$$\ddot{\boldsymbol{x}} = -\boldsymbol{\nabla}\Phi, \tag{9}$$

where for MOND $\boldsymbol{\nabla}\Phi = \boldsymbol{g}$ is calculated from equation (4).

Equation (9) represents a set of coupled differential equations that we solve numerically using the fourth-order Runge–Kutta technique (Press et al. 1992) with a time-step of 0.15 Myr. Reducing the time-step was found to produce converged results, while, for purely spherical potentials, the code was found to conserve energy and angular momentum to machine accuracy (better than 1 part in 10^7).[3]

4 RESULTS

We modelled the Sagittarius dwarf orbit by fitting to the best-fitting orbit presented in Law et al. (2005) and ensuring that the current position and velocity of the dwarf matched observational constraints. This gives a current phase-space position and velocity of the dwarf (in Galactocentric *right-handed* coordinates) of $x = 16.2$, $y = 2.3$, $z = -5.9$, $v_x = 238$, $v_y = -42$, $v_z = 222$, in units of kpc and km s^{-1} respectively. This satellite phase-space position was then integrated backwards 1 Gyr in time to match the trailing arm of the Sagittarius dwarf and forwards 1 Gyr in time to match the leading arm. Unlike Law et al. (2005), we did not allow the final phase-space position of the dwarf to vary within observational constraints, but held this fixed. This is because we wish to measure the difference in orbital precession between the models, which is easier to do if the initial phase-space coordinates are identical.

Fig. 2 shows orbital projections for all five models as shown in the legend, integrated over 2 Gyr. The Galactic plane (not marked) is perpendicular to the z-axis. The top panels are for the best-fitting Sagittarius dwarf orbit, which is on a near-polar orbit around the Galaxy (Law et al. 2005). The bottom panels are for a small-pericentre orbit, which samples a wide range of the Galactic potential. The position of the Sagittarius dwarf is now marked on the top panels with a solid square. The orbital pole precession is shown in the right-most panels. This measures the difference in angle between the vector perpendicular to the satellite's orbit initially and at a given time.

All of the models produced very similar orbits for the Sagittarius dwarf, and the differences are best seen in the right-most-panel plots of the orbital pole precessions. The pole precession is a useful quantity to measure for the orbits because it is a strong function of how flattened a potential is – this is why the differences between the orbits in each of the models show up so strongly in the plot of the pole precession, whereas they are much harder to detect in the plots of the orbital projections. Small differences in the orbits between models can be accounted for by altering the details of the *visible* component of the Milky Way potential (recall that the Kuzmin disc used for the MOND model is only an approximation to the true potential of the Milky Way disc and bulge), or by altering the final phase-space position of the Sagittarius dwarf within observational constraints as done by Law et al. (2005). The pole precession, however, can only be reproduced by changing how flat the potential is. In the CDM models, this can be achieved quite easily by using a more oblate dark matter halo. In the MOND model there is less freedom to do this.

[3] The orbits in the axisymmetric potentials used in this paper also conserved energy and the z-component of the angular momentum to machine accuracy – as expected for potentials with axisymmetry (Binney & Tremaine 1987).

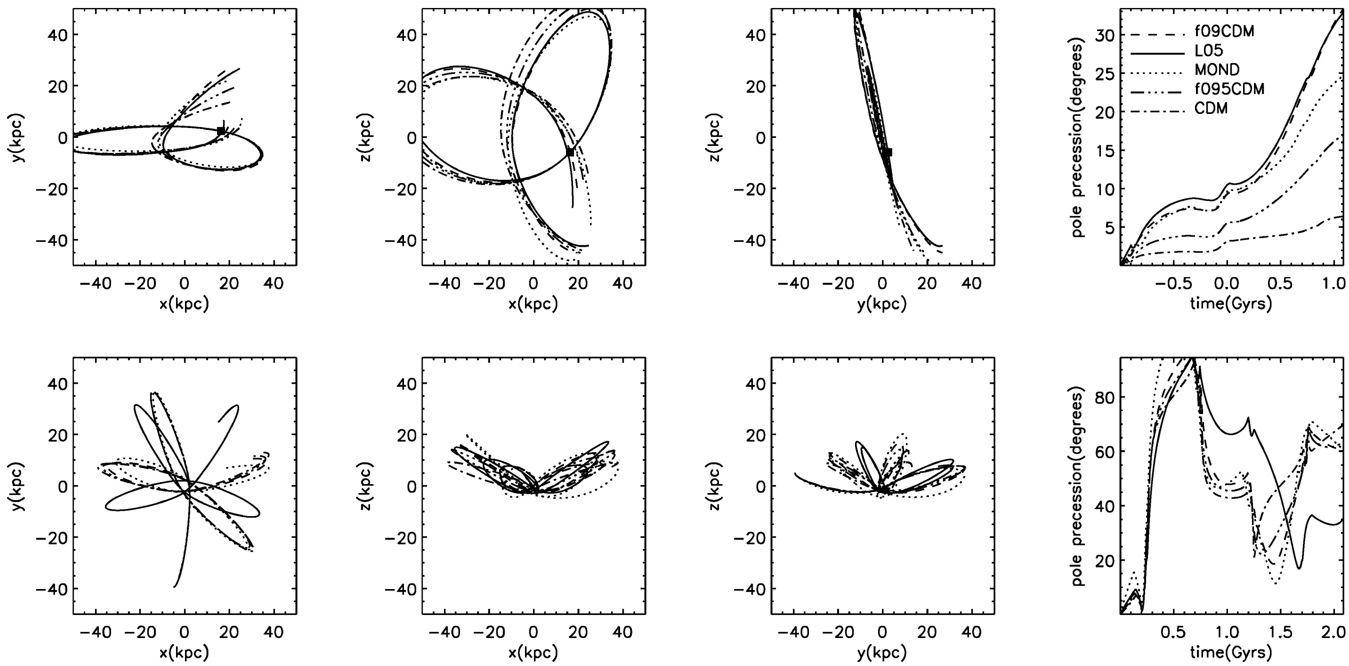

Figure 2. Orbital projections for five models as shown in the legend, integrated over 2 Gyr. The Galactic plane (not marked) is perpendicular to the *z*-axis. The top panels are for the best-fitting Sagittarius dwarf orbit, which is on a near-polar orbit around the Galaxy (Law et al. 2005). The bottom panels are for a small-pericentre orbit that samples a wide range of the Galactic potential. The position of the Sagittarius dwarf now is marked on the top panels with a solid square. The orbital pole precession is shown in the right-most panels. For the Sagittarius dwarf orbit, the time marked is relative to its current phase-space position. The observed trailing arms then trace out the orbital path that the dwarf took over the past Gyr (hence the *negative* time), while the leading arms show what its path will be over the next Gyr (hence the positive time).

While changing the mass of the Milky Way disc can produce more or less precession, there are strong limits from stellar population models and from the rotation curve of the Milky Way on the extent to which this can be done. If the MOND model produces far too much, or far too little precession as compared with the Sagittarius stream, we can rule it out as a viable alternative theory to dark matter.

The CDM model (dot-dashed line) produced the least precession, as expected for a near-spherical potential (recall that this model still contains a massive disc and so we should still expect some precession). The f09CDM and L05 models produced very similar results. This is because, with an orbital pericentre greater than 10 kpc, the Sagittarius dwarf is not sampling a region of the Milky Way potential where the presence of the bulge is significant. The key point is that, surprisingly, the MOND model (dotted line) produced *near-identical pole precession* to both the L05 and f09CDM models – consistent with the best-fitting orbit for the Sagittarius dwarf debris. If anything, the MOND model produced *too little* precession for the leading arm of the Sagittarius dwarf debris. This result seems surprising since in the MOND model all of the gravity is coming from the disc. We will discuss this further in Section 5.

There could, then, be an early indication that the MOND model is inconsistent with the Sagittarius stream orbit. However, current data from the Sagittarius dwarf are only good enough to constrain the pole precession over the period -0.4 to 0.6 Gyr with an error of 2°–3° (Johnston et al. 2005). This places a flattening of $q = 0.9$–0.95 within the 1.5σ error bars. Yet the difference in pole precession between $q = 0.9$ and $q = 0.95$ is much larger than the difference between the MOND and the f09CDM or L05 models. Over the range -0.4 to 0.6 Gyr discussed in Johnston et al. (2005), the MOND model produces near-identical results to the f09CDM and L05 models.

There is still a lively debate about what the best-fitting orbit for the Sagittarius dwarf actually is (see Section 4.1). However, it seems unlikely that it would be possible to produce *more* precession in MOND than that from an infinitesimally thin disc (the potential used in this paper). If this is true, then it may be possible in the future to rule out MOND on the grounds that it *does not produce enough* precession to match the Sagittarius stream data.

The bottom panels show a more extreme orbit. Notice that now the orbit for the L05 model strongly deviates from the others. This is because this satellite orbit has a pericentre of $\sim$2 kpc and so the satellite is now sampling the region of the Milky Way potential where the bulge makes a difference. The rest of the models, which have no bulge component, once again show very similar orbits. The pole precession in the MOND model is again best matched by that in the f09CDM model, while the f095CDM and CDM models show less precession, as is expected from their rounder halo potentials.

The MOND potential produces orbits that are very similar to those in a CDM halo with flattening of $q \sim 0.9$. This seems to be the case even for orbits with unrealistically small pericentres. This suggests that, for the Milky Way at least, it will be difficult to distinguish between a MOND and a dark matter model for the Galaxy using satellite streams. Even if better data could be obtained for the globular cluster orbits (which orbit closer to the Galaxy), MOND can be expected to produce results similar to those for a mildly oblate dark matter halo.

4.1 The Sagittarius leading-arm velocity data

There has been some debate in the literature over the velocity data for the Sagittarius dwarf leading arm. We have so far in this paper referred only to the spatial data from the Sagittarius stream and not to the velocity data.

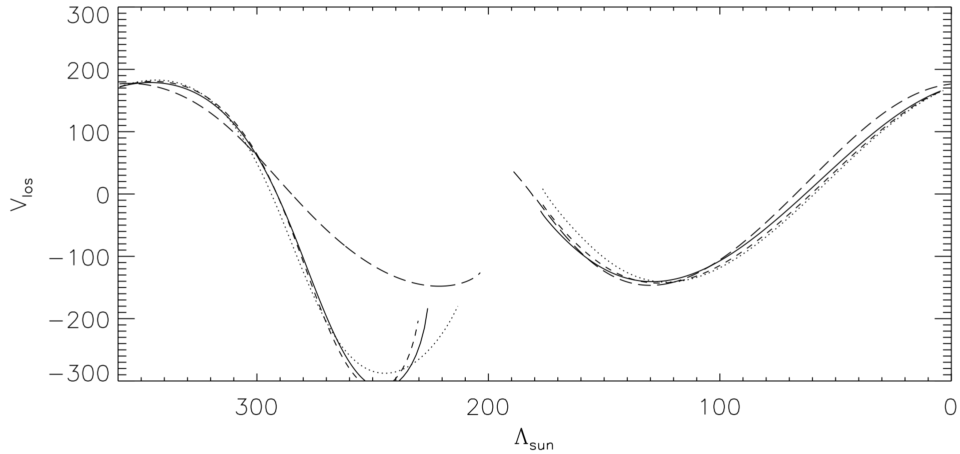

Figure 3. Line-of-sight velocity of the Sagittarius stream as viewed from the Sun as a function of its longitude on the sky. The lines for the models are as previously: MOND is dotted, L05 is solid and f09CDM is dashed. The extra line shown is for a *prolate* CDM model (long-dashed line) with $q = 1.25$. It is the long-dashed line that provides the best fit to the velocity data for the Sgr stream, particularly for the leading-arm data (left-most curves on the plot). Notice that the MOND model (dotted line) agrees well with the L05 and f09CDM models as before (dashed and solid lines), but that all of the models differ greatly from the prolate model.

Helmi (2004) has recently pointed out that the leading-arm velocity data are inconsistent with the halo flattening of $q = 0.9$ advocated by Johnston et al. (2005) and suggest that a prolate halo ($q \sim 1.25$) would provide a better fit if the velocity data are taken into account. However, Johnston et al. (2005) argue that a prolate halo leads to an incorrect value for the orbital pole precession; in fact, a prolate halo causes the orbit to precess in the *opposite* direction to that observed. They point out that it is difficult to obtain the correct amount of pole precession without altering the underlying potential (as can be seen in the pole precession plots in Fig. 2), whereas one could conceivably alter the velocities of the stars in the plane of the stream through second-order effects such as dynamical friction (Law et al. 2005). Could MOND perhaps reconcile the discrepant leading-arm velocity data?

Fig. 3 shows the the line-of-sight velocity of the Sagittarius stream as viewed from the Sun as a function of its longitude on the sky. The lines for the models are as previously: MOND is dotted, L05 is solid, and f09CDM is dashed. The extra line shown is for a *prolate* CDM model (long-dashed line) with $q = 1.25$. It is the long-dashed line that provides the best fit to the velocity data for the Sgr stream, particularly for the leading-arm data (left-most curves on the plot). Notice that the MOND model (dotted line) agrees well with the L05 and f09CDM models as before (dashed and solid lines), but that all of the models differ greatly from the prolate model.

MOND does not solve the problem of the discrepant leading-arm velocity data for the Sagittarius stream. As with the spatial data for the stream, MOND produces a near-identical orbit to the L05 and f09CDM models. If it can be shown that the Milky Way halo (or any other galaxy halo) must be prolate, this, as pointed out by Helmi (2004), would be difficult to reconcile with MOND.

5 DISCUSSION

5.1 Model assumptions

Perhaps the biggest assumption in this work is the highly specific choice of potential for the Galaxy disc. While this could be problematic for a detailed study of satellite orbits in the Milky Way, we have implicitly considered the case of maximal precession in MOND. It would be difficult to imagine a more axisymmetric potential than an infinitesimally thin disc. This does mean, then, that should a dark matter halo be discovered that is significantly more oblate than $q = 0.9$, this would be difficult to reconcile with MOND.

We have, furthermore, neglected dynamical friction and not performed a detailed N-body simulation of the stripping of stars from the Sagittarius dwarf galaxy. As such, this work should not be taken as conclusive evidence that the Sagittarius stream is consistent with MOND.

Finally, we should note that MOND is probably the best-studied but not the only alternative gravity theory (see e.g. Drummond 2001; Moffat 2005). Some of these theories predict a Keplerian fall off at large radii in the rotation curve, similar to in CDM models (see e.g. Moffat 2005). These may be even more difficult to rule out using tidal streams.

5.2 Why does such a flat mass distribution in MOND produce such a round potential?

This point has been discussed in some detail in Milgrom (2001), but is perhaps best illustrated by direct integration of equation (4) for a Kuzmin disc. In the deep-MOND limit, $\mu(x) \rightarrow x$ and we find from equations (4) and (5) that

$$\Phi \simeq \frac{(MGa_0)^{1/2}}{2} \ln(R^2 + (|z| + a)^2), \tag{10}$$

which is very nearly identical to the flattened logarithmic halo (see equation 6). Thus, highly flattened *mass distributions* in MOND do not produce highly flattened potentials. In fact, it is the spherical nature of the Kuzmin potential in the deep-MOND limit that leads to MOND producing slightly too little precession in the Sagittarius orbit (see Fig. 2). From Fig. 1, we can see that the MOND rotation curve (dotted line) is flat beyond ∼20 kpc, indicating that it is then in the deep-MOND limit. The other rotation curves for the CDM models are all falling at these radii rather than being flat – a property that cannot be achieved in MOND, since it is a theory set up to produce flat rather than falling rotation curves at large radii. Thus, interior to ∼20 kpc, all of the models agree quite well in their rotation curves, and the corresponding orbit for the Sagittarius *trailing* arm looks very similar. This can be seen in the good agreement from −1 to 0 Gyr in the orbital pole precessions for the f09CDM, L05 and

MOND models (Fig. 2, top right panel). However, the leading-arm orbit – which can be seen in the pole precession plot from 0 to 1 Gyr – does not agree so well. For this part of the orbit, the Sagittarius dwarf moves out towards apocentre and samples the region of the potential where the Milky Way is fully in the MOND regime and where the rotation curve (in MOND) is flat and near-spherical.

5.3 Exploiting flattened elliptical galaxy potentials

Buote & Canizares (1994) and Buote et al. (2002) have recently shown that the observed flattening of hot X-ray gas in elliptical galaxies can also be used to place tight constraints on MOND. They argue that, if the hot gas in NGC 720 is in hydrostatic equilibrium, $\nabla p_{\mathrm{gas}} = -\rho\nabla\Phi$, which implies that $\nabla\rho \times \nabla\Phi = \mathbf{0}$. Thus, the X-ray isophotes from the hot gas in NGC 720 trace the gas density, which in turn traces the underlying gravitational potential. By deprojecting the stellar potential they show that the stars in NGC 720 cannot produce a flat enough potential in MOND to produce the observed X-ray isophotes.

While they have to assume hydrostatic equilibrium and some simple form for the deprojected stellar potential, their assumptions are quite conservative. They find that MOND produces potentials that are too spherical at large radii – similar to what *may* be the case here for the leading-arm Sagittarius dwarf debris.

5.4 What are the prospects for constraining MOND using tidal streams?

Cold dark matter haloes, as modelled in *N*-body cosmological simulations, are typically 2:1 triaxial systems. This would make the Milky Way with $q \sim 0.9$ quite rare. However, recent simulations by Kazantzidis et al. (2004) have shown that the dissipation of baryons changes the inner regions of galactic haloes to be nearly spherical and consistent with the flattening predicted from the Sagittarius stream. The amount of flattening depends sensitively on the fraction of baryons that undergoes slow dissipation to form the galactic disc.

MOND mimics haloes with $q \sim 0.9$, while a CDM cosmology produces similar haloes as a result of gas cooling and galaxy formation. This will make it difficult to differentiate between MOND and CDM theories using halo flattening, even with a large statistical sample of halo shapes.

6 CONCLUSIONS

We have compared orbits in a thin axisymmetric disc potential in MOND with those in a thin disc plus near-spherical dark matter halo predicted by ΛCDM cosmology. We have demonstrated that the amount of orbital precession in MOND is very nearly identical to a similar CDM galaxy with a logarithmic halo with flattening $q = 0.9$, consistent with recent constraints from the Sagittarius stream. Since very flattened mass distributions in MOND produce more spheroidal potentials than in standard Newtonian mechanics, we have shown that it will be very difficult to use the tidal debris from streams to distinguish between a MOND galaxy and a standard CDM galaxy with a mildly oblate halo.

If a galaxy can be found with either a prolate halo or one that is more oblate than $q \sim 0.9$ this would rule out MOND as a viable theory. Improved data from the leading arm of the Sagittarius dwarf – which samples the Galactic potential at large radii – could rule out MOND if the orbital pole precession can be determined to an accuracy of the order of $\pm 1^{\circ}$.

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